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A STUDY FOR THE FAST SOLUTION OF ELECTROMAGNETIC SCATTERING PROBLEMS: A WAVELET BASED APPROACH

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ABSTRACT

Conventional method of moments (MoM), when directly applied to integral equations arising in numerical solution of electromagnetic(EM) scattering problems , leads to a dense(fully populated) matrix which often becomes computationally ungovernable even for supercomputers, especially when the electrical size of the scatterer becomes large. To overcome this difficulty, wavelet bases have been used recently which lead to a sparse matrix that can be solved easily by an efficient sparse solver. Using wavelet in solving EM integral equations has been widely studied. The purpose of this paper is to develop a strategy for efficient wavelet solution of integral equations by connecting and using efficient studies have been done in this area. Numerical results are provided to illustrate the validity of the proposed approach.

INTRODUCTION

A large class of EM scattering problems can be formulated by the following integral equation

$$\int f(s')G(s,s')ds' = g(s) \tag{1}$$

where f(s) stands for the induced surface current, $G(s,s')$ is the Green's function, and g(s) stands for the excitation source. Generally, equation (1) has no closed-form solutions and the MoM is used to solve it numerically. As well known, the use of traditional basis and testing functions for solving in the MoM results in a dense matrix equation. A direct solution of a dense matrix equations needs $O(N^3)$ operations, and an iterative solutions requires $O(N^2)$ operations per dense matrix-vector multiplication, where N is the number of unknowns in the discretized integral equations. Therefore, traditional MoM is not of practical use, as the number of unknowns increases, due to the large memory requirement and high computation time necessary to solve matrix equation.

To overcome these difficulties, recently, EM researchers used wavelets, primarily because of their local supports and vanishing moment properties, to solve EM integral equations. There are currently two approaches to introducing wavelets in the MoM: In the first, the integral equation has been directly expanded and tested with wavelet bases functions [1]-[2]. However, since few kinds of wavelets can be solved in closed form, this approach requires considerable numerical effort to efficiently evaluate the integrals, which dims the use of wavelets in the MoM. The other approach is to use a conventional basis and testing functions to convert integral equation into matrix equation and then perform a discrete wavelet transform on the resultant matrix equation [3]. More recently, the authors have proposed an effective circulant wavelet transform method, which can be adaptively used to solve efficiently a wide range of EM problems [4]. In this study, therefore, we use the approach used in [4].

FORMULATION

By using the MoM, we obtain the matrix equation

$$ZI = V \tag{2}$$

where Z is a dense impedance matrix. Introducing a wavelet matrix W, the matrix equation in (2) then transformed as

$$Z'I' = V' \tag{3}$$

where

$$Z' = WZW^T, \quad I' = (W^T)^{-1}I, \quad V' = WV. \tag{4}$$

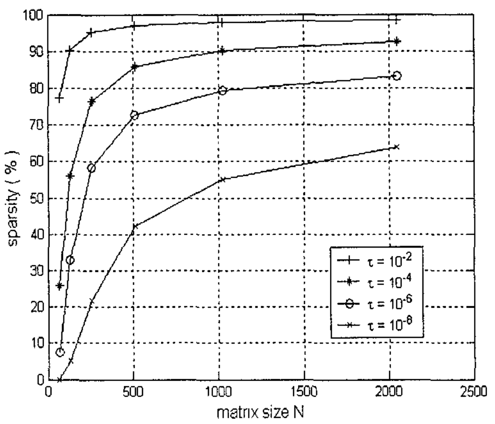
Here T stands for the transpose of a matrix. For a given threshold value τ ($0 < \tau < 1$), (3) becomes a sparse matrix equation which can be efficiently solved by a sparse solver. Once I' is solved, the desired solution is obtained as

$$I = W^T I' \tag{5}$$

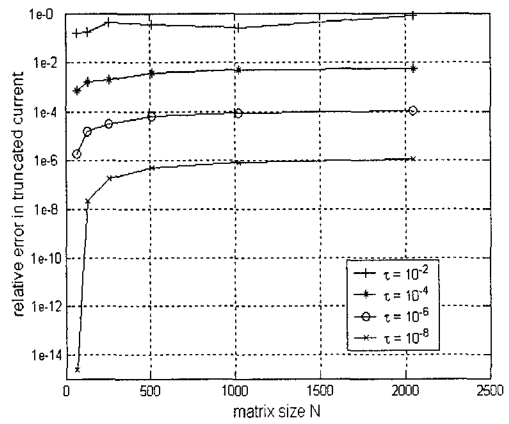
The construction of the wavelet matrix W can be found in[4]. In constructing W, among the wavelet types, Daubechies' wavelet is chosen, because of its compactness and orthogonality properties, to effectively construct sparse wavelet matrix, which reduces the computational, cost [5]. Finally, an appropriate choice of the number of vanishing moments of wavelets is made as 8 from [4] to obtain fast and accurate solution in the numerical experiments.

NUMERICAL RESULTS

In this section, the results of a study of matrix sparsity as a function of the problem size are presented. Scattering of plane wave from 2-D rectangular cylinder is computed numerically using a constant number of test functions (20 pulse) per wavelength. The system sizes studied ranged from N = 64 (contour length of 3.2 λ) to N = 2048 (contour length of 102.4 λ). The sparsity of truncated Z' and the associated relative error of current distribution on the contour surface for several thresholds is shown in Fig. 1. Here the percent sparsity is $S = ((N_0 - N_\tau)/N_0)x100$ where N_0 is the total number of elements and N_τ is the number of remaining element after the truncation. The relative error caused by the truncation is $\epsilon = \|I_0 - I_W\|_2 / \|I_0\|_2$ where I_0 is the solution obtained by the MoM and I_W is that obtained from the wavelet method.



(a)



(b)

Fig. 1 Matrix Sparsity(a) and Relative Error in Current(b) as a Function of Size N

CONCLUSION

The EM scattering from a 2-D rectangular cylinder has been successfully analyzed by using the wavelet matrix transform approach. Numerical results have shown that the present approach does highly sparsify MoM matrices, especially as the problem size increases, and hence dramatically reduces the computation time by a efficient sparse solver without causing much error in the solutions.

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